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# A comparison of missing data handling methods in linear structural relationship model: evidence from BDHS2007 data

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Missing observations in dependent variable is a common feature in survey research. A number of techniques have been developed to impute missing data. In this article, we have evaluated the performance of several imputation methods namely mean-before method, mean-before-after method and expectation-maximization algorithm in linear structural relationship model. On the basis of mean absolute error and root mean square error for both simulated and real data sets, we have shown that expectation-maximization algorithm is the most effective method than the other two imputation methods to analyze the missing data in linear structural relationship model.

**keywords:** Errors-in-variable model, Imputation method, Expectation-maximization algorithm, Performance indicator, Demographic health survey.

## 1 Introduction

Missing data are a part of almost all research and it has a negative influence on the analysis, such as information loss, as a result, a loss of efficiency, loss of unbiasedness of

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estimated parameters and loss of power. There are several reasons why the data may be missing. They may be missing because of equipment malfunctioned, the data were not entered correctly, sometimes respondent skip questions out of fear of getting in trouble or a subject may be removed from a trial if his/her condition is not controlled sufficiently well. Based on the different reasons, missing data can be classified as missing completely at random (MCAR), missing at random (MAR), and non-ignorable (NI) missing values (Little and Rubin, 2002) and these classifications influence the optimal strategy for working with missing values.

Several studies (Hippel, 2004; Winkler and McCarthy, 2005; Little and Rubin, 2002) have been conducted on how to deal the data set with missing values. Imputation methods are one of the most widely used technique to solve incomplete data problem (Little and Rubin, 2002). Therefore, this study stresses on several imputation methods to determine the best methods to replace missing data.

From a complete data set, incomplete data sets need to be generated in order to compare different methods. Many researchers (Junninen et al., 2004; Twisk and Vente, 2001; Norazian et al., 2008) generated randomly simulated missing data patterns for evaluating different methods in different types of data sets, as for example, air quality data sets and longitudinal studies. Perneger and Burnand (2005) applied simple imputation algorithm to reduced missing data in SF-12 health surveys. Moreover, Kofmanhttp (2000) applied the imputation method for incomplete dependent variables in the financial data set.

In this article, we generate different missing data patterns in dependent variable and apply three imputation methods on Bangladesh Demographic and Health Survey 2007 data (NIPORT et al., 2009) namely the mean-before, mean-before-after and expectation-maximization (EM) algorithm for the parameters of linear structural relationship model (LSRM) assuming the ratio of error variance is known. The performance of imputation methods is measured using mean absolute error and root mean square error. We organize this article as follows: Section 2 gives materials and methods which includes the brief description of the estimation of parameters in LSRM, mean imputation techniques, expectation-maximization algorithm, performance indicators, simulation studies and description of the data BDHS 2007 data. Section 3 envelops the results and discussion of the study. Finally, in Section 4, a conclusion is given.

## 2 Materials and Methods

### 2.1 Estimation of parameters in LSRM

Consider the following circumstances

$$Y = \alpha + \beta X \quad (1)$$

where, there exists a linear relationship between the random variables  $X$  (heights) and (weights) and suppose that they are measured without error.

However, in reality, these two variables  $X$  and  $Y$  are not observed directly, i.e., they are measured subject to error. If  $\delta_i$  and  $\epsilon_i$  are the two respective errors in measuring  $X_i$

and  $Y_i$ , then we can write  $x_i = X_i + \delta_i$  and  $y_i = Y_i + \epsilon_i$ , where the error terms  $\delta_i$  and  $\epsilon_i$  are normally distributed having zero mean and variance  $\sigma_\delta^2$  and  $\sigma_\epsilon^2$ , respectively. This reveals that the variances of error are not dependent on  $i$  and so independent of the level of  $X$  and  $Y$ , which assumed homoscedasticity. There are some assumptions that have been described in the literature for obtaining the  $X$  values. For example, Kendall and Stuart (1973) described the structural model considering  $X_i$  as normal distribution with mean  $\mu$  and variance  $\sigma_X^2$ . In LSRM, the errors are assumed to be normal, the bivariate normal distribution of  $x_i$  and  $y_i$ , is then

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} \sim N \left( \begin{bmatrix} \mu \\ \alpha + \beta\mu \end{bmatrix}, \begin{bmatrix} \sigma_X^2 + \sigma_\delta^2 & \beta\sigma_X^2 \\ \beta\sigma_X^2 & \beta^2\sigma_X^2 + \sigma_\epsilon^2 \end{bmatrix} \right) \quad (2)$$

Kendall and Stuart (1973) have shown that there are five equations with six unknown  $(\mu, \alpha, \beta, \sigma_X^2, \sigma_\delta^2, \sigma_\epsilon^2)$ , hence an additional assumption is required for the unique and consistent solutions of the parameters of the model (1). In particular, Hood et al. (1999) discuss in detail estimation procedure to estimate the model (1) under various assumptions. However, for the case when the ratio of error variance  $\lambda = \frac{\sigma_\epsilon^2}{\sigma_\delta^2}$  is assumed to be known, the Maximum Likelihood Estimate (MLE) for the parameters are given by

$$\hat{\mu} = \bar{x} \quad (3)$$

$$\hat{\alpha} = \bar{y} + \hat{\beta}\bar{x} \quad (4)$$

$$\hat{\beta} = \frac{(S_y^2 - \lambda S_x^2) + \sqrt{(S_y^2 - \lambda S_x^2)^2 + 4\lambda S_{xy}^2}}{2S_{xy}} \quad (5)$$

$$\hat{\sigma}_X^2 = \frac{S_{xy}(\hat{\beta}^2 S_y^2 + 2\lambda\hat{\beta}S_{xy} + \lambda^2 S_x^2)}{(\hat{\beta}^2 + \lambda)(\hat{\beta}S_y^2 + \lambda S_{xy})} \quad (6)$$

$$\hat{\sigma}_\delta^2 = \frac{(\hat{\beta}S_x^2 - S_{xy})(\hat{\beta}^2 S_y^2 + 2\lambda\hat{\beta}S_{xy} + \lambda^2 S_x^2)}{(\hat{\beta}^2 + \lambda)(\hat{\beta}S_y^2 + \lambda S_{xy})} \quad (7)$$

where,  $S_x^2$ ,  $S_y^2$  and  $S_{xy}$  are defined as  $S_x^2 = \frac{1}{n} \sum (x_i - \bar{x})^2$ ,  $S_y^2 = \frac{1}{n} \sum (y_i - \bar{y})^2$  and  $S_{xy} = \frac{1}{n} \sum (x_i - \bar{x})(y_i - \bar{y})$ , respectively.

Recently, Carpita and Ciavolino (2015) proposed generalized maximum entropy (GME) estimator for a simple linear measurement error model with a composite indicator. Further, they (Carpita and Ciavolino, 2016) developed GME estimator for the regression model with a composite indicator as explanatory variable.

## 2.2 Mean Imputation Techniques

Let us consider  $n$  observations  $x_1, x_2, \dots, x_n$  of which  $m$  values are missing denoted by  $x_1^*, x_2^*, \dots, x_n^*$ . Thus, the observed data with missing values are (Ayyub and McCuen, 1996)

$$x_1, x_2, \dots, x_{n_1}, x_1^*, x_{n_1+1}, x_{n_1+2}, \dots, x_{n_2}, x_2^*, x_{n_2+1}, x_{n_2+2}, \dots, x_m^*, x_n \quad (8)$$

Therefore, the first missing value occurs after  $n_1$  observations, the second missing value occur after  $n_2$  observations, and so on. Note that there might be more than one consecutive missing observation. The mean-before-after technique substitutes all missing values with the mean of one datum before the missing value and one datum after the missing value. Thus for the data in (8),  $x_1^*$  will be replaced by (Yahaya et al., 2005)

$$\bar{x}_1 = \frac{x_{n_1} + x_{n_1+1}}{2} \quad (9)$$

and  $x_2^*$  will be replaced by

$$\bar{x}_2 = \frac{x_{n_2} + x_{n_2+1}}{2} \quad (10)$$

and so on. The mean-before technique substitutes all missing values by the mean of complete case data only (without missing values). Thus for the data in (8),  $x_1^*$  and will be replaced by (Yahaya et al., 2005)

$$\bar{x}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i \quad (11)$$

and  $x_2^*$  will be replaced by

$$\bar{x}_2 = \frac{1}{(n_2 - n_1 - 1)} \sum_{i=n_1+1}^{n_2} x_i \quad (12)$$

and so on.

### 2.3 EM-algorithm

The EM-method is chosen to be the imputation method because the properties of EM itself will cause the likelihood keep on increasing and this makes EM numerically stable. Other than that, EM usually handles parameter constraints automatically since each M-step produces MLE-type estimates. Discussion on EM algorithm can be found in Dempster et al. (1977). Applications of EM algorithm in many areas especially in handling an incomplete data were carried out by Geng et al. (2000); Gaetan and Yao (2003); Wu (1983); Sundberg (1974) and others.

Basically, there are two steps in the EM algorithm which can be called as the Expectation or E-step and Maximization or M-step (Dempster et al., 1977).

**(a) E-step:** Finding the expected value of the complete data likelihood given the observed data  $Y$  and initial parameter estimation,  $\Theta^{i-1}$ .

$$Q(\Theta, \Theta^{i-1}) = E[\log p(X, Y | \Theta) | X, \Theta^{i-1}]$$

**(b) M-step:** In this step, the expectation that calculated in E-step will be maximized. These two steps will be repeated necessarily until they converge to a local maximum of the likelihood function. A discussion on the convergence properties for EM algorithm can be found in Wu (1983).

## 2.4 Performance Indicator

We considered three performance indicators; say, mean absolute error (MAE) and root mean square error (RMSE) to examine the imputation methods. In order to select the best method for estimating missing values, the predicted and observed data are compared.

The mean absolute error is the average difference between predicted and actual data values (Chen et al., 1998), and is given by

$$MAE = \frac{1}{N} \sum_{i=1}^N |P_i - O_i| \quad (13)$$

where,  $N$  is the number of imputations,  $P_i$  and  $O_i$  are the imputed and observed data points, respectively. The MAE varies from 0 to infinity and perfect fit is obtained when  $MAE = 0$ .

The root mean squared error is one of the most commonly used measure (Junninen et al., 2004), which is given by

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N |P_i - O_i|^2} \quad (14)$$

The smaller is the RMSE value, the better is the performance of the method.

## 2.5 Simulation Study

In this section, we have carried out a simulation study in order to investigate the performance of the three imputation methods. For this experiment, we consider the parameter settings ( $\alpha = 0, \mu = 10, \beta = 1, \sigma_X^2 = 5, \sigma_\delta^2 = 0.50, \sigma_\epsilon^2 = 1$ ) and sample sizes  $n = 50$  and  $70$ . To select the value of  $\sigma_X^2$ , we follow the principle of Hood et al. (1999) in which the difference caused by measurement errors will be dominated by the difference between the mean levels. The different levels of missing values say, 5%, 10%, 20% and 30% are inserted randomly in the dependent variable. Furthermore, on the basis of MAE and RMSE in 10,000 trials; we examine the properties of these three methods.

## 2.6 Description of the Data

Bangladesh Demographic and Health Survey (BDHS) 2007 (NIPORT et al., 2009) is a nationally representative sample survey designed to provide information on basic national indicators of social progress including fertility, childhood mortality, contraceptive knowledge and use, maternal and child health, nutritional status of mothers and children, awareness of AIDS, and domestic violence.

A total of 10,859 women age 15-49 and 3,771 men age 15-54 from 10,400 households cover 361 sample points (clusters) throughout Bangladesh, 134 in urban areas and 227 in the rural areas. The characteristic of weights (dependent variable) and heights (independent variable) of women are given in Table 1 and Table 2, respectively. In order

Table 1: Descriptive statistics of dependent variable (weights) for BDHS 2007 data

Number of valid data points	10859
Number of missing data points	137
Mean	47.4825
Mode	42.2
Standard Deviation	9.2938
Skewness	1.0031
Kurtosis	1.5422
Percentile	25      40.9
	50      45.9
	75      52.5

Table 2: Descriptive statistics of independent variable (heights) for BDHS 2007 data

Number of valid data points	10859
Number of missing data points	137
Mean	150.5553
Mode	150.5
Standard Deviation	6.5356
Skewness	-4.4108
Kurtosis	70.1384
Percentile	25      146.8
	50      150.5
	75      154.2

to make the relationship (1), we assume that both variables contain measurement error. For examining the accuracy of imputation techniques, we randomly generated 5%, 10%, 20%, 25% and 30% missing data points in dependent variable  $Y$  of the complete BDHS 2007 data set (data set without its own 137 missing data points).

### 3 Results and Discussion

In this section, we compare the different methods of missing values based on simulation study and BDHS2007 data results.

#### 3.1 Comparison of the Different Methods of Missing Values based on Simulation Study

Table 3 demonstrates that for the sample of size  $n = 50$  at 5% missing values, the EM gives MAE=1.8541 and RMSE= 2.1572 respectively, whereas the mean-before (1.8795 and 2.3844) and the mean-before-after (2.2426 and 2.4920) method give bigger MAE and RMSE values than EM from the simulated data set. Again, when the sample size remains same but the percentage of missing values increase to 10%, 20% and 30%, the EM gives the smallest values of MAE and RMSE than the other two methods. It can be noted that the EM provides almost same values of MAE and RMSE at the different percentage of missing values. However, for the same sample size ( $n=50$ ), the value of MAE and RMSE increase as the percentage of missing values increase for the methods of mean-before and mean-before-after.

Table 4 also represents the simulation envelope for the various percentage of missing values for the fixed sample size  $n=70$ . Like as the simulation results of Table 3, Table 4 shows that the value of MAE and RMSE for EM method are always smaller than the value of the mean-before and mean-before-after method. Again when the percentage of missing values increases, the values of MAE and RMSE for EM differs slightly but the value of MAE and RMSE for the mean-before and the mean-before-after method increases drastically. Moreover, when the sample size varies from 50 to 70 with fixed percentage of missing values (say 10%), the MAE and RMSE of the EM gives almost similar results, whereas it is not true for the mean-before and the mean-before-after methods. A similar conclusion can be made from other percentage of missing values.

Finally, from Table 3 and Table 4, we can say that EM produces the smallest MAE and RMSE than the other two imputation methods for different percentage of missing values and for different sample sizes. It is also observed that the values of MAE and RMSE using the EM method are fairly close with increasing the percentage of missing values. The mean-before and the mean-before-after method, however, show larger values of MAE and RMSE with increasing the percentage of missing values.



Table 3: The MAE and RMSE from three imputation methods for  $n = 50$ 

Percentage of Missing Data	Methods	MAE	RMSE
5%	Mean-before	1.8795	2.3844
	Mean-before-after	2.2426	2.4920
	EM	1.8541	2.1572
10%	Mean-before	4.8886	3.6819
	Mean-before-after	5.6892	4.2681
	EM	1.8723	2.2330
20%	Mean-before	10.3717	5.6520
	Mean-before-after	11.3848	6.1647
	EM	1.8894	2.3061
30%	Mean-before	16.1996	7.2737
	Mean-before-after	17.1567	7.6558
	EM	1.8979	2.3414

Table 4: The MAE and RMSE from three imputation methods for  $n = 70$ 

Percentage of Missing Data	Methods	MAE	RMSE
5%	Mean-before	1.9707	2.3237
	Mean-before-after	2.2949	2.7125
	EM	1.8726	2.2076
10%	Mean-before	3.4467	3.1519
	Mean-before-after	3.9889	3.6382
	EM	1.8759	2.2715
20%	Mean-before	7.2998	4.8004
	Mean-before-after	7.9690	5.2315
	EM	1.8793	2.3126
30%	Mean-before	12.2200	6.4382
	Mean-before-after	12.5435	6.5955
	EM	1.9732	2.3913

### 3.2 Comparison of the Different Methods of Missing Values based on BDHS 2007 data

From the simulation study, we get an impression that the EM algorithm outperforms than the other two existing methods for linear structural related variables. The same phenomenon is checked in this section by a practical demographic survey data, BDHS 2007 data. The descriptive statistics for BDHS 2007 data set are shown in Table 5 in which different percentage of missing values are generated. The mean values vary slightly with the percentage of missing data points. In spite of differences in the amount, it is interesting that the analysis generates similar types of results for all percentage of missing values. It can also be seen that there is very small variation in the percentiles with the respective percentage of missing values. This is due to the way in which the missing values were generated, and to the occurrence of a large number of observations within the same range. From Table 6, EM algorithm gives smallest MAE and RMSE for all types of simulated missing data patterns than the other two methods.

Table 5: Descriptive statistics of the dependent variable (weights) for BDHS 2007 data

Percentage of Missing data	5%	10%	15%	20%	25%	30%
Number of valid data points	10316	9773	9230	8687	8144	7601
Number of missing data points	543	1086	1629	2172	2715	3258
Mean	47.4481	47.4847	47.4088	47.5035	47.4329	47.5045
Standard Deviation	9.3044	9.2971	9.2976	9.2883	9.3239	9.2807
Skewness	1.0201	0.9821	1.0012	0.9599	1.0416	1.0009
Kurtosis	1.6125	1.4027	1.5230	1.3096	1.6596	1.5896
Range	76.80	74.80	76.80	74.80	73.40	76.8
Minimum value	24	24	24	24	25.4	24
Maximum value	100.8	98.80	100.80	98.80	98.80	100.80
Percentile	25	40.90	40.90	40.80	40.90	41.0
	50	45.85	45.90	45.80	45.90	45.9
	75	52.40	52.50	52.40	52.50	52.5

Table 6: The MAE and RMSE from three imputation methods of the dependent variable (weights) for BDHS 2007 data

Percentage of Missing Data	Properties	Mean-before	Mean-before-after	EM
5%	MAE	7.8962	8.2780	7.2182
	RMSE	9.8594	10.5343	9.0899
10%	MAE	7.2132	7.6706	7.0976
	RMSE	9.3126	9.9255	9.2642
15%	MAE	7.1044	7.7106	7.0452
	RMSE	9.4095	10.0475	9.2738
20%	MAE	7.2664	7.6499	7.1381
	RMSE	9.4537	9.8569	9.3159
25%	MAE	7.6102	7.7259	7.1752
	RMSE	9.9262	9.9053	9.2033
30%	MAE	7.8359	7.6255	7.2577
	RMSE	10.1830	9.8703	9.3242

## 4 Conclusions

In this article, different imputation methods are compared to handle the missing data in linear structural relationship model. Two performance indicators namely MAE and RMSE are used to find the most effective imputation method. Based on the simulated data set, it is observed that the EM method performs better than the other two imputation methods to impute missing data in LSRM as it produces the smallest MAE and RMSE. It is also observed that EM method produces relatively same values for MAE and RMSE, whereas the mean-before and mean-before-after method show an increasing pattern in MAE and RMSE when the percentage of missing values are increased. Similar conclusions can be made for real BDHS2007 data set that the EM performs better than the other two imputation methods.

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